The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—7

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. A particle moves along the X-axis in such a way that its acceleration at time t for $t > 0$ is given by $a(t) = \frac{3}{t^2}$. When $t = 1$, the position of the particle is 6 and the velocity is 2.

   (a) Write an equation for the velocity, $v(t)$, of the particle for all $t > 0$. $-\frac{3}{t}+5$

   (b) Write an equation for the position, $x(t)$, of the particle for all $t > 0$. $-3\ln t+5t+1$

   (c) Find the position of the particle when $t = e$. $5e-2$

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2. Given that f is the function defined by $f(x) = \frac{x^3 - x}{x^3 - 4x}$.

   (a) Find $\lim\limits_{x \to 0} f(x)$. $\frac{1}{4}$

   $x \neq 0$ (b) Find the zeros of f. $x = \pm 1$

   $x \neq 0$ (c) Write an equation for each vertical and each horizontal asymptote to the graph of f. $x = \pm 2$, $y = 1$

   (d) Describe the symmetry of the graph of f. y axis

   (e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f on the axes provided.

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3. Let R be the region in the <u>first quadrant</u> that is enclosed by the graph of $y = \tan x$, the X-axis, and the line $x = \frac{\pi}{3}$.

   (a) Find the area of R. $\ln 2$

   (b) Find the volume of the solid formed by revolving R about the X-axis. $\pi\left(\sqrt{3}-\frac{\pi}{3}\right)$

TURN THE PAGE

$x^2+y^2=15^2$

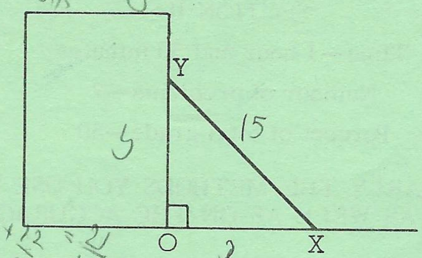
$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$2(9)(0.5)+2(12)\frac{dy}{dt}=0$

$\frac{dy}{dt}=-\frac{9}{24}=-\frac{3}{8}$

$=-0.375$

$\frac{dx}{dt}=\frac{1}{2}$



$A=\frac{1}{2}xy$

$\frac{dA}{dt}=\frac{1}{2}x\frac{dy}{dt}+\frac{1}{2}y\frac{dx}{dt}$

$=\frac{1}{2}(9)(-\frac{3}{8})+\frac{1}{2}(12)(\frac{1}{2})$

$=-\frac{27}{16}+\frac{12}{4}=\frac{21}{16}$

$9^2+y^2=15^2$

$y^2=144$

$y=12$

4. A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building. $-\frac{3}{8}$

(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building. $\frac{21}{16}$

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5. Let f be the function defined by $f(x) = (x^2 + 1)\,e^{-x}$ for $-4 \leqq x \leqq 4$.

(a) For what value of x does f reach its absolute maximum? Justify your answer.

(b) Find the x-coordinates of all points of inflection of f. Justify your answer.

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6. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base and \$5 per square meter for the sides, what is the cost of the least expensive tank?

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7. For all real numbers x, f is a differentiable function such that $f(-x) = f(x)$. Let $f(p) = 1$ and $f'(p) = 5$ for some $p > 0$.

(a) Find $f'(-p)$.

(b) Find $f'(0)$.

(c) If ℓ_1 and ℓ_2 are lines tangent to the graph of f at $(-p, 1)$ and $(p, 1)$, respectively, and if ℓ_1 and ℓ_2 intersect at point Q, find the x- and y-coordinates of Q in terms of p.

5) $f(x)=(x^2+1)e^{-x}$ $\quad -4\le x\le 4$

$f'(x)=-e^{-x}(x^2+1)+e^{-x}(2x)=0$

$-e^{-x}(x^2-2x+1)=0$

$-e^{-x}(x-1)(x-1)=0$

$e^{-x}\neq 0$

$x=1$

END OF EXAMINATION

$f''(x)=e^{-x}(2x-2)-e^{-x}(x^2-2x+1)=0$

$e^{-x}(2x-2-x^2+2x-1)=0$

$e^{-x}(-x^2+4x-3)=0$

$-e^{-x}(x^2-4x+3)=0$

$-e^{-x}(x-1)(x-3)=0$

$x=1, x=3$

$f'(x)$

-4 $\quad 1 \quad 4$

$f(-4)=e^{4}(17)=17e^{4}$

$f(4)=17e^{-4}=\frac{17}{e^4}$

$f(1)=\frac{2}{e}$